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Robustness of pure white pseudonoise signal to temporal fluctuation in impulse response measurement

Kentaro Mori and Yutaka Kaneda*

Graduate School of Engineering, Tokyo Denki University,
5 Senju-Asahi-cho, Adachi-ku, Tokyo, 120–8551 Japan

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1. Introduction

In impulse response measurement, the linear time invariance of transmission systems is assumed. Therefore, measurement errors arise when there is a temporal fluctuation in transmission systems caused by wind during the measurement in a large space or a minute lag of clocks between a digital-to-analog (DA) reproducing system and an analog-to-digital (AD) recording system. The magnitude of the errors depends on the type of measurement signal. The magnitude of the errors is small when a swept sine signal, such as a time-stretched pulse (TSP), is used; however, it is large when a pseudonoise signal, such as a maximum length sequence (M-sequence), is used [1,2]. However, pseudonoise has a major advantage that it reduces the effect of an impulsive noise. Therefore, in this study, we examine the cause of errors when a pseudonoise signal is used and propose a new signal called a “pure white pseudonoise signal” [3] to reduce the errors.

2. Errors caused by temporal fluctuation

Figure 1 shows a schematic of the measurement of the frequency characteristics (quantity equivalent to the impulse response) of a transmission system $H(k)$. Here, k is the discrete frequency number and is omitted in the figure. In the absence of temporal fluctuation, $H(k)$ can be determined regardless of the type of measurement signal $S(k)$. In contrast, in the presence of temporal fluctuation, the result contains an error and is given by $H'(k)S'(k)/S(k)$. Here, $H'(k)$ and $S'(k)$ represent the frequency characteristics and the measurement signal subjected to temporal fluctuation, respectively.

As explained in Sect. 1, the magnitude of the errors strongly depends on the type of measurement signal. Therefore, the cause of errors is not the change in $H(k)$ but the deviation of $S'(k)/S(k)$ from 1. When a white signal, such as an M-sequence signal, is assumed as the measurement signal,

$$\left|\frac{S'(k)}{S(k)}\right| = |S'(k)| \tag{1}$$

holds because $|S(k)| = 1$ for any k ($k = 0, 1, 2, 3, \ldots$). When focusing on the amplitude spectrum of the measurement, the errors are considered to be caused by the deviation of $|S'(k)|$ from 1.

Here, part of the interpolated amplitude spectrum of an M-sequence [obtained by a discrete Fourier transform (DFT) of a zero-padded M-sequence signal] is shown as a solid line in Fig. 2. In Fig. 2, the open circles represent the original discrete spectrum when the time axis in the M-sequence signal is not expanded or contracted. The amplitude at these discrete frequency points is constant regardless of the frequency. The open squares represent the discrete amplitude spectrum when the frequency axis is contracted due to the time axis expansion caused by the clock time mismatch between DA and AD converters. When the analogue M-sequence signal is represented by $m(t)$, the time axis expansion is represented as $m(a \cdot t)$, where $a < 1$ is a positive constant. In this case, the amplitude fluctuates significantly. In contrast, a TSP signal has a flat interpolated spectrum and, hence, is less affected by temporal fluctuation.

As explained, pseudonoise signals with a nonflat interpolated spectrum are easily affected by temporal fluctuation. In this study, we propose a new pseudonoise signal with a flat interpolated spectrum.

3. Pure white pseudonoise signal

Figure 3 shows the algorithm for synthesizing a pure white pseudonoise signal.

① An M-sequence signal (or a discrete white pseudonoise signal) of length L, $w(n)$ (n, discrete time), is used as the original signal.

② A sufficient number of zeros (here, $9L$ zeros) are added to $w(n)$ as padding, which is then subjected to a DFT to obtain an interpolated spectrum.

③ Because the amplitude spectrum obtained (interpolated) in step ② fluctuates as shown by the solid line in Fig. 2, the amplitude spectrum is forcibly made to be 1 over the entire frequency to make it a white signal spectrum while maintaining the phase spectrum.

④ When a time waveform is obtained by the inverse DFT of the whitened amplitude spectrum obtained in step ③, a nonzero amplitude is observed at sampling points longer than L, as shown in Fig. 4.

⑤ The nonzero amplitude is forcibly made to be 0 to correct the signal length to L.

⑥ The DFT of the corrected signal is carried out. Although the characteristics of a complete white signal are lost as a result of step ⑤, the amplitude spectrum is closer to

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*e-mail: kaneda@c.dendai.ac.jp

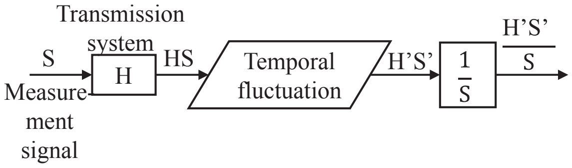

Fig. 1 Schematic of frequency characteristic measurement of a transmission system with temporal fluctuation.

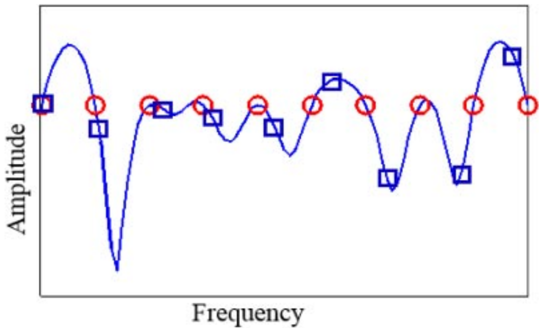

Fig. 2 Part of discrete amplitude spectrum and interpolated spectrum of M-sequence signal.

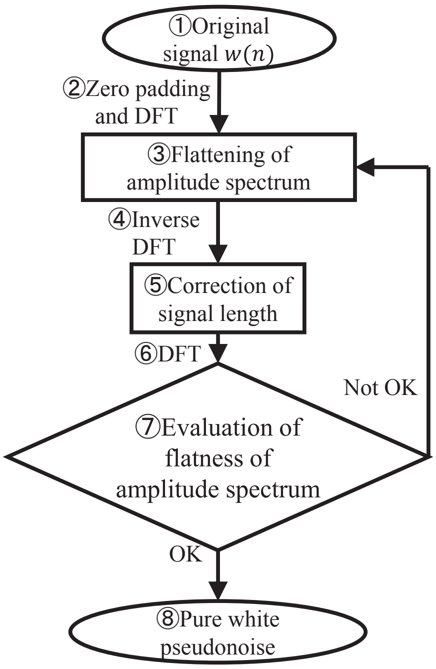

Fig. 3 Algorithm for synthesizing pure white pseudonoise signal.

the spectrum of white signals than the initial amplitude spectrum.

⑦ The whiteness (flatness) of the spectrum is evaluated. The mean amplitude spectrum in dB is determined and the maximum of the deviation $\varepsilon_{\max}$ from the mean amplitude spectrum is used as the evaluation value. Steps ③–⑥ are repeated until sufficient whiteness is obtained, i.e., $\varepsilon_{\max}$ is lower than a threshold value (in this study, $\varepsilon_{\max} < 0.05$ dB).

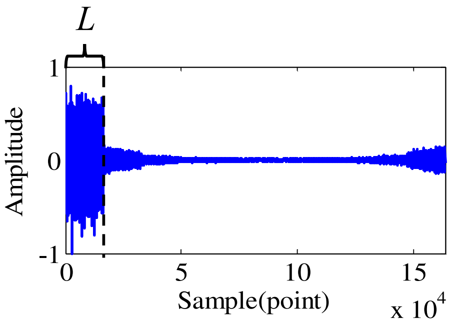

Fig. 4 Time waveform obtained by inverse DFT of flattened amplitude spectrum.

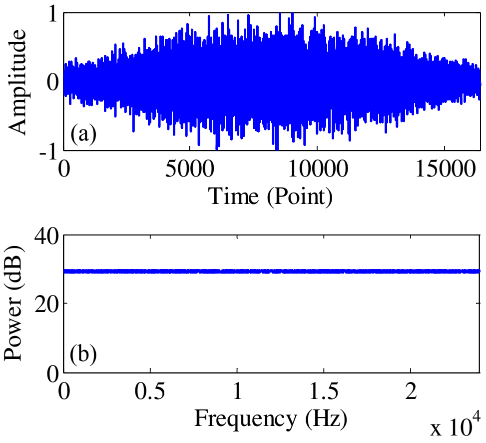

Fig. 5 (a) Time waveform and (b) interpolated amplitude spectrum of pure white pseudonoise.

⑧ The process is completed when the above conditions are satisfied. Figure 5(a) shows the time waveform of the obtained pure white pseudonoise signal. The amplitude of the signal at both ends converges to a small value. Figure 5(b) shows the amplitude spectrum interpolated by applying zero padding ($9L$ zeros) and a DFT to the signal shown in Fig. 5(a). The amplitude spectrum is an almost white spectrum even after interpolation.

4. Simulation experiment

4.1. Simulation conditions

To confirm the robustness of a pure white pseudonoise signal to temporal fluctuation, a simulation was carried out. The lengths of an M-sequence signal and a pure white pseudonoise signal were $2^{14} - 1$ samples and the sampling frequency was 48 kHz. The transmission system was assumed to have a flat amplitude and a linear phase frequency characteristic (i.e., a pure-delay characteristic). Then, the time axis was uniformly expanded or contracted so that the entire length of the measurement signal ($2^{14} - 1$ samples) was increased or decreased by 0.5 samples.

4.2. Simulation results

Figures 6(a) and 6(b) show the amplitude–frequency characteristics of the objective transmission system obtained by the simulation using (a) an M-sequence signal and (b) a

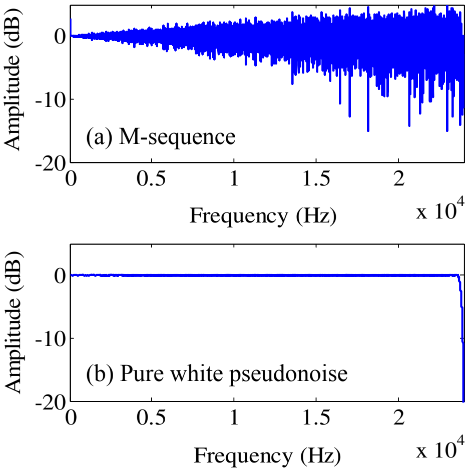

Fig. 6 Frequency characteristics obtained when temporal fluctuation is applied for (a) M-sequence and (b) pure white pseudonoise.

pure white pseudonoise signal when the time axis was expanded. As shown in Fig. 6(a), when the temporal fluctuation is applied to the M-sequence signal, the amplitude more greatly fluctuates, or includes more errors, as the frequency increases.

In contrast, when the temporal fluctuation is applied to the pure white pseudonoise signal, the amplitude is almost constant over the entire frequency range, as shown in Fig. 6(b). Thus, in the case of the expansion of the time axis, resulting in the contraction of the frequency axis (rightward movement of the sampling point), the amplitude spectrum obtained using a pure white pseudonoise signal is less affected and almost constant over all frequencies.

When the contraction of the time axis was applied, a result similar to that shown in Fig. 6 was obtained (data not shown). From the above findings, we showed that the robustness of the pure white pseudonoise signal to temporal fluctuation is superior to that of the M-sequence signal.

5. Conclusions

In this study, we proposed a pure white pseudonoise signal that maintains spectral whiteness even after interpolating the complex white spectrum defined at discrete frequencies. Conventional white pseudonoise signals, such as an M-sequence signal, contain large errors when there is a temporal fluctuation in the measurement system. We showed, by simulation, that the pure white pseudonoise signal proposed in this study is negligibly affected by temporal fluctuation.

Acknowledgement

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References

[1] H. Nakajima, K. Takashima, H. Tsuru and S. Ohhashi, "A wind effect on impulse response measurement using TSP and M-sequence," *Proc. Autumn Meeting Acoust. Soc. Jpn.*, 1-9-2, pp. 473–474 (1998) (in Japanese).

[2] S. Müller and P. Massarani, "Transfer-function measurement with sweeps," *J. Audio Eng. Soc.*, **49**, 443–471 (2001).

[3] K. Mori and Y. Kaneda, "Generation method of pure white noise for impulse response measurement," *Proc. 2015 IEICE General Conf.*, A-10-2 (2015) (in Japanese).